

GEOMETRY
NO FRIEND to
INFIDELITY:
OR, A
DEFENCE
OF

Sir ISAAC NEWTON

AND THE

British Mathematicians,

In a LETTER to the Author of the *Analyst*.

Wherein it is examined

How far the Conduct of such Divines as intermix the Interest of Religion with their private Disputes and Passions, and allow neither Learning nor Reason to those they differ from, is of Honour or Service to Christianity, or agreeable to the Example of our Blessed Saviour and his Apostles.

By PHILALETES CANTABRIGIENSIS.

*No Deus interfit, nisi dignus vindice nodus
Inciderit.*

L O N D O N:

Printed for T. COOPER at the *Globe* in *Ivy-Lane*.
M DCC XXXIV. Price 1 s.

GLENN

OF THE

STATE OF

NEW YORK

IN SENATE

January 10, 1890

REPORT

OF THE

COMMISSIONERS

OF THE LAND OFFICE

IN RESPONSE TO A RESOLUTION

PASSED BY THE SENATE

ON JANUARY 10, 1890

ALBANY:

W. H. BROWN, PRINTERS

1890

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ERRATA.

Pag. 22. line 8. for *of* read *in*. p. 56. l. ult. dele *an*. p. 59.
l. 15. r. and *me*. p. 60. l. 1, r. *him*?



LETTER

To the AUTHOR of the

ANALYST.

Cambridge, April 10. 1734.

SIR,

AS I am one of those many persons in this University, who have profited by your learned writings, and who greatly admire the depth of thought, the force of reason, and the perspicuity of expression that generally appears in them; I cannot but be extremely surprized and concerned, that a Gentleman of your abilities should have taken so much pains not only to depreciate one of the noblest of all the sciences, but to disparage, to traduce, and even to defame a set of learned men, whose labours so greatly conduce to the honour of this island, and to the general good of man-

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kind.

kind. You will easily see that I speak of Mathematicks, and the *British* Mathematicians, and of the manner in which you have been pleased to treat them in your late pamphlet, intituled, *The Analyst*. That discourse, it is true, is seemingly directed to one person only; but you are not got through your first section, before you speak of *too many more of the like character* with him; and in the contents to that very section, as well as in all the rest of your discourse, you talk of Mathematicians in so general a manner, that I am afraid your readers will think very few, if any, of those Gentlemen are to be excepted out of that number, against which you bring so heavy a charge.

This charge, Sir, consists of three principal points.

1. Of infidelity with regard to the Christian Religion.
2. Of endeavouring to make others Infidels, and succeeding in those endeavours by means of *the deference* which is paid to *their judgment*, as being presumed to be of all men the greatest masters of reason.
3. Of error and false reasoning in their own science.

This,

This, Sir, is the charge you have been pleased to bring against our Mathematicians, and I intend particularly to examine with what reason and truth you have done so. But before I do this, I must beg leave to make a few observations upon the intent and design of your treatise, the advantage to be expected from it, the prudence, and justice and honour of this design; and after that to inquire whether the motive to your writing it were really what you pretend and avow.

If your design were to be guessed at from your Title-page, wherein you profess to *examine, whether the object, principles and inferences of the modern Analysis are more distinctly conceived, or more evidently deduced, than religious mysteries and points of faith*, one would be apt to conclude your intent was to shew, that the mysteries of Religion might be as clearly conceived, as the object and principles of the modern Analysis; and that the several points of our Faith might be as evidently deduced, as the inferences of that Analysis are from its principles; or, in other words, that you were about to give us a Mathematical demonstration, or one of equal clearness and certainty, of the truth of the

Christian Religion. This, I say, from the words of your Title-page one would naturally take to be your design: but it is so far from being so, that throughout your whole discourse, though addressed to an *Infidel Mathematician*, I do not find the least attempt towards establishing the evidence of our Christian Faith.

I find therefore no other way of reconciling your Title-page with the substance of your discourse, than by supposing you pretend to prove, that the object, principles, and inferences of the modern Analysis are not more distinctly conceived, nor more evidently deduced, than religious Mysteries and point of Faith, *i. e.* that there is no more evidence and certainty in the modern Analysis, than in the Christian Religion. But how far you do honour to Christianity, by entering into a comparison between the evidence and certainty of the Religion taught us by our Saviour and his Apostles, and the evidence and certainty of the doctrine of Fluxions delivered by Sir *Isaac Newton* and his followers, a doctrine in your opinion, *full of error, false reasoning, manifest sophisms, fallacious and inconsistent ways of arguing, such as would not be allowed of in Divinity,*

I shall

I shall leave to your brethren the Divines to consider.

Not to insist therefore too long upon this, I shall now dismiss your Title-page, and shall come to your declared and avowed design, which, it must be owned, you have steadily pursued through your whole discourse; to lessen the reputation and authority of Sir *Isaac Newton* and his followers, by shewing that they are not such *great masters of reason*, as they are generally presumed to be; and to depreciate the science they profess, by demonstrating to the world that it is not of that clearness and certainty, as is commonly imagined.

You must excuse us, Sir, if this design appears a very strange one to us of the University, who plainly see of how great use Mathematical Learning is to mankind, not only to those who make it a part of their studies, but to all the rest of the world, who without knowing any thing of Mathematics, do yet daily and hourly reap the benefit of the inventions of Mathematical men in all parts of life, especially in Mechanical arts, in Architecture, Civil, Naval, and Military, and in Navigation, upon which the prosperity and security of this Nation so
 4 much

much depends. Though we see and know, that the study of Mathematicks flourishes among us, as much as in any part of the world; and that our youth have as good assistance and opportunities for cultivating that science, as are any where to be met with; yet we are far from thinking that too many of our students engage in this pursuit, or spend too much of their time in it. Believe me, Sir, the generality of youth have more need of the spur than the bridle, when they are to enter upon a study, that requires so laborious an attention, and so constant an application. Why then, would we ask, is the study of Mathematicks to be discouraged and undervalued? What could be the reason or motive for your engaging in such a design?

I perceive your answer is ready. Mathematicians are Infidels, and make use of their reputation and authority, as being esteemed *the greatest Masters of reason*, to pervert other persons to infidelity. Really, Sir, this assertion of yours is new to us, and extremely surprizes us. We see nothing of it here, and cannot easily believe it is so any where else.

But

But admitting it for the present to be true; have you well considered whether it be for the advantage of Christianity, to publish to the world, that a numerous set of learned men, and of such learned men as are ** supposed to apprehend more distinctly, consider more closely, infer more justly, conclude more accurately than other men*, do not believe the truth of the Christian Religion? This, methinks, is an assertion fitter for an *Alci-phron*, or a *Lyficles* to make, than for a Christian Divine. *Tell it not in Gath, publish it not in the streets of Ascalon.* Though the evidence and certainty of our holy Religion is so firmly established, as not to be shaken by the arguments, much less by the reputation and authority of any unbelievers whatsoever; yet, I am afraid, it would be a great stumbling block to men of weak heads, if they were made to believe, that the justest and closest reasoners were generally Infidels. You know, Sir, it was a shrewd objection, tho' by no means a valid or conclusive one, that was made against our Saviour. *Have any of the High Priests, or of the Pharisees* (the men of greatest learning, and esteemed the greatest masters of rea-

son in those days) *believed in him?* And if there were any room to renew this objection in our days, as considering the piety and zeal of the Christian Priests of all degrees, and their constant and sincere attachment to the interest of the Christian Religion and to nothing else, surely there is not, it would indeed be a very perplexing one.

If we should now make you a farther concession, and suppose it not only to be true, but to be publicly and notoriously known, that the body of our Mathematicians are Infidels and enemies to Christianity, what think you is the method to be taken? What other, you will say, but to ruin the reputation of our adversaries? If we cannot attack them in their lives and conversations, we must at least endeavour to shew that they are destitute of learning and reason. This will lessen their authority among the people, will prevent their ** misleading unwary persons in matters of the highest concernment*, and will take off *† that bias and deference for their judgment*, which causes weak *‡ minds to submit to their decisions, where they have no right to decide.* Why really, Sir, this may do very well, if you are sure

** Analyst, p. 3.*

† p. 4.

‡ Ibid.

of succeeding in your attempt. At least it must be acknowledged, that you do *stare super vias antiquas*. This is the very method which the *Odium Theologicum*, the intemperate zeal of Divines has always pursued, and has practised with great success for many ages. But I must beg leave to observe, that it is a very different course from what was taken by *Jesus Christ* and his Apostles on the like occasion. *I thank thee, O Father, Lord of heaven and earth,* says our Saviour, *that thou hast hid these things from the wise and prudent, and hast revealed them unto babes.* And after his example, the great Apostle of the Gentiles, though he was fully sensible of the opposition he met with in preaching the Gospel, from the *Greek Philosophers*; yet did not think it necessary to enter the lists with the Disciples of *Plato* and the *Stoicks*, and to shew that they did not understand Geometry or Philosophy; in order to lessen their reputation, and thereby to take off from the weight of their opposition to Christianity. He contents himself with observing, that *not many wise, not many learned are called*; though *the Greeks seek after wisdom*, though *Christ crucified is unto the Greeks foolishness*,

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yet *the wisdom of this world is foolishness with God.* Many more passages may be found in his Epistles to the same purpose.

You see, Sir, that neither our Saviour, nor his Apostles, who were guided by his Holy Spirit, denied either wisdom, or prudence, or learning to such unbelievers as were really possessed of those qualities: they well knew there was nothing to fear, that *there was no enchantment against Jacob, nor divination against Israel*, and that no human abilities or accomplishments whatsoever could be able to withstand the Divine authority, and the irresistible evidence of the Religion they taught and defended.

It will be said perhaps, this is too great a pattern for you to follow; though there was no fear that the abilities, the reputation and authority of any adversaries whatsoever could withstand our Saviour or his Apostles; yet what was no match for them, may possibly be too hard for you, unless you take proper means to prevent it. How, Sir! Has not our Saviour promised *to be with you unto the end of the world?* Has he not assured us, that *even the gates of hell shall not prevail against his Church?* After this declaration,

claration, is there any thing to fear? Can any means be necessary, but such as he has recommended by his practice and example? Can calumny, and detraction, and artifice to lessen the reputation of your adversaries, be means fit to be used by the followers of *St. Paul* and of *Jesus Christ*? If any human means are requisite, surely they should be such as are innocent and just, rather than this criminal method of lessening or detracting from your opponents.

What are these? you will say. Why plainly, Sir, to fight your enemies with their own weapons, to endeavour to excel in those arts with which they oppose you. *St. Paul made himself all things to all men, if thereby he might gain some.* If Mathematicians are such dangerous adversaries to Christianity, let the Church of *England* take care of the education of her Clergy, let her write over her pulpits, as *Plato* is said to have done over the entrance of his school, *εἰς ἀγεομέτρητος εἰσίτω*, Let no man enter into orders, unless he be an able Mathematician. When this is done, Sir, let us see what Mathematical Infidel will dare to beard a Christian Priest, as *great a master of reason* as himself, and armed besides with his

Theological accomplishments, and the authority of his sacred Function, but above all with that ineffable and incomprehensible Gift, which every one of you receive at your ordination.

Surely no man will think this too laborious a task for the zeal and piety of the Christian Clergy to undertake, if the salvation of souls depends upon it. Do not we see how wholly they devote themselves to this only end and purpose; how, neglecting and despising all the gratifications of sense, all the allurements of worldly interest, of honour, riches and power, they allot every portion of their time that can possibly be spared from the necessary exercises of their function, to the better enabling themselves to take care of the souls committed to their charge? Do not they for this purpose, with incredible pains and indefatigable industry, ransack and make themselves masters of all the treasures of antiquity sacred and profane? Hence those nervous and cogent arguments, that invincible power of reason, that resistless force of eloquence which thunders from every pulpit, to withstand the gainfayer, to reclaim the deluded, to confirm the wavering, to silence, confound
and

and astonish the obstinate, incorrigible Heretic and Infidel. And shall we, can we suppose, that a small part of their time and pains will not be allowed for arming themselves with proper weapons to encounter our Mathematical unbelievers? You, Sir, have already shewn them how easily this is to be done; since notwithstanding your many and great avocations, both at home and in very distant regions, you have not only made your self master of the most profound, the most obscure and most difficult parts of that study; but by looking closer and more attentively into them than any body has ever done before you, have discovered their most secret defects, and have unravelled their most hidden and intricate fallacies. And I make no doubt but your great example will excite such a spirit of piety and zeal, and industry among your brethren, that in time to come even *Euclid*, and *Archimedes*, and *Apollonius*, if they are judged to stand in the way and obstruct the progress of Christianity, will be shewn to abound as much with false and fallacious reasoning, as Sir *Isaac Newton*.

This, Sir, would be the innocent, the just and honourable method of encountering the adversaries of our Religion; rather to meet

meet them in the field, than to seek to nail their cannon, to blow up their ammunition, and set fire to their magazines: but if you chuse the other, as safer, though less commendable, as easier to your brethren, though more laborious to your self, I am content; and provided you go through with the undertaking, shall be ready to join with the Reverend Defenders of our faith, in a solemn chorus to your eternal honour and renown.

*Unus homo nobis carpendo restituit rem:
Non ponebat enim rumores ante salutem:
Ergo magisq; magisq; viri nunc gloria claret.*

Now therefore, Sir, we come to our main and principal point, to examine by what means, in order to prevent the danger that Christianity is threatened with from our Mathematical Infidels, you propose to lessen their reputation, and to shew that they are not such just and close reasoners as is commonly imagined. I shall prove, say you, that the method of Fluxions invented by Sir *Isaac Newton*, and implicitly received by all our Mathematicians, is built upon false and precarious principles.

This

This will do something, I must confess, in as much as it will shew, that Mathematicians are men and liable to error. But if they are mistaken in thinking the method of Fluxions to be a sound and just one, will it follow from this, that in all the other parts of their science they do not reason justly and accurately? You know very well, Sir, that Fluxions are of very late invention, and that though they are a noble and useful part of Mathematicks, yet the whole of Mathematicks does not consist in Fluxions. Mathematicians were always reckoned to be *great masters of reason*, before Fluxions were invented. And as the use of Fluxions does by no means supersede the doctrine of Geometry delivered by *Euclid*, *Apollonius*, and so many others of the great masters both antient and modern, not to mention Algebra and all the other parts of the Mathematicks, against which you make no objection; will it not follow that Mathematicians are still to be esteemed just and accurate reasoners, though they have been led into some mistakes in the method of Fluxions, by their implicit deference to the judgment of the Great Author of that invention?

But

But farther, if it should appear that there is no mistake in that method, that the object of it is clearly conceived, that its principles are evident and certain, that the inferences drawn from them are justly deduced, that the whole doctrine is established upon a clear, just, and solid foundation, and that it is you only who have been guilty of a most inexcusable oversight, in which the meanest Mathematician you could have consulted, would have immediately set you right: what then will become of your noble and well concerted project to lessen the reputation of Mathematicians for the service of Christianity?

Believe me, Sir, our holy Religion stands in need of no such attempts to serve it. *Non tali auxilio, nec defensoribus istis.* The better reasoners either Mathematicians, or any other sort of men shall be, the more likely will they be to embrace truth wherever it shall be found, and consequently the more likely to receive the truth and undoubted evidence of the Christian Religion.

But pray, Sir, was it really and indeed the interest of Christianity, and nothing else, that prompted you to this undertaking? Was that your only motive? The reason of my doubt is,

is, that Sir *Isaac Newton*, who by your own acknowledgment was not an Infidel, is no more spared in your performance, nay on the contrary is more severely handled than any of his followers, whom you suppose to be Infidels. For besides the charge brought against him and them in common, of the *want of good Geometry and good Logic, of error and false reasoning*, you frequently represent him as *conscious of the defect of his principles*, and yet endeavouring to impose upon and to mislead his followers *by shifts, artifice and subtilty, by ambiguous terms, by puzzling, palliating and fallacious ways of proceeding, &c.* that is, you represent them only as weak men, deceived and imposed upon; but him both as a weak and an ill man; as not only deceived, but a deceiver and an impostor. Did zeal for Christianity move you to this? To treat in this manner a person whom you acknowledge to have been a believer of the Christian Religion? I am afraid we must look out for some other motive, and in order to discover it, I shall take the liberty of considering a little what has been your conduct with regard to Mathematicians, not only since you have been informed of their being Infidels, but five and twenty years ago, when

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it may be reasonably supposed you had heard and seen less of them, than you have lately done.

About that time you published an *Essay towards a new Theory of Vision*, wherein you were pleased to insert a great many severe censures upon Mathematicians, relating to their ignorance of the fundamental principles of their own science. Had you then heard of their being Infidels? If so, why were many of those censures greatly mollified, upon farther consideration, in a new edition of that piece published the last year? And how comes a new charge of ignorance to be brought against them, on another account, at this time? Really, Sir, I can no way ascribe all this to a zeal for Christianity: but, if I may be allowed to make one hypothesis, it will then be easy to account clearly and distinctly for every particular of your conduct. It is briefly this, that you have too great an opinion of your self, and too mean a one of all other men. Hence, not content with the reputation you have deservedly acquired of being a clear and just reasoner, you can never rest, unless you convince the world that all those, who have hitherto been esteemed the *greatest masters*
of

of reason, are in this respect greatly inferior to Dr. B—y. Else why these attacks upon Mathematicians in general? Why are Dr. *Barrow* and Sir *Isaac Newton* singled out from among the Mathematicians; and why Mr. *Locke* and Lord *Shaftsbury* among those who are esteemed clear and just reasoners, though not Mathematicians, but to shew that Dr. B—y is a closer, a juster, and more accurate reasoner than any of these four Gentlemen not to be matched perhaps by four others, not only in *Great Britain*, but in the world? Were these Gentlemen all unbelievers? Yourself will not call them so; and if we except one, you will not in the least suspect them to be so. Why then are they all attacked in your writings? I cannot possibly surmise any other cause, but that they are all esteemed *great masters of reason*, and that you are disposed to lessen their reputation in order to exalt your own. Two of them you have been pleased to charge with contradiction, gross, apparent, glaring contradiction, such as were utterly unpardonable in a man who makes the least pretence to reasoning; such as no body could possibly suspect in those great men, who had not a much greater opinion of his own abilities

than of theirs; such as no man who had the least distrust or diffidence of himself, could ever imagine them to be guilty of. I would say, Sir, that had you supposed it possible for your self ever to fall into any mistake; you would certainly, when you imagined Sir *Isaac Newton* and Mr. *Locke* to have so manifestly contradicted themselves, have looked over those passages of their writings a second time; you would have done this coolly, attentively and considerately; and had you done so, you would have found your self in a gross, evident mistake; and would have seen them to be clearly and perfectly consistent with themselves. I must take leave to add, that your not doing so, does evidently prove the truth of the hypothesis I laid down but now, that you have too good an opinion of your self, and too mean a one of those great men. One of these I shall of course be led to vindicate in the pursuit of my design; and as your mistake about the other has a near relation to Geometry, before I close this letter, I shall take up a little of your time in rectifying that likewise.

Having finished my considerations upon the design, the usefulness, prudence and justice

stice of your performance, and your motive for entering upon it ; I come now to examine into the charge you have brought against our Mathematicians.

1. Of infidelity with respect to the Christian Religion.
2. Of endeavouring to make others Infidels, and succeeding in those endeavours, by means of *that deference* which is paid *to their judgment, as being presumed to be of all men the greatest masters of reason.*
3. Of error and false reasoning in their own science.

Here, Sir, I must beg leave to ask, what proof you can bring of the two first articles of this accusation? I have carefully perused your whole discourse, and do not find the least foundation for them, except only that you say, in your first page, *you are not a stranger to the authority that one nameless Infidel Mathematician assumes in things foreign to his profession, nor to the abuse that he, and too many more of the like character, are known to make of such undue authority, to the misleading of unwary persons in matters of the highest concernment, and whereof his Mathematical knowledge can by no means qualify him to be a*
competent

competent judge ; and that in your second page, you tell us, That this is one way of making Infidels, you are credibly informed.

How say you, Sir, *you are not a stranger to it, it is known, you are credibly informed? Bene habet, nil plus interrogo.* Let us burn or hang up all the Mathematicians in *Great Britain*, or halloo the mob upon them to tear them to pieces every mother's son of them, *Tros Rutulusve fuat*, Laymen or Clergymen, tho' some of these, by their preaching, their writings, their lives and conversations, are thought to do honour to their order. Let us dig up the bodies of *Dr. Barrow* and *Sir Isaac Newton*, and burn them under the gallows, and demolish the monuments erected to their memories. What, tho' the first be esteemed one of the greatest luminaries of the Christian Church, and the other be acknowledged to have been a true believer, and to have given some of the strongest and clearest proofs, that have ever been produced, of the goodness, wisdom, and power of the Supreme Being? No matter: their followers are Freethinkers, Minute Philosophers and Infidels, and labour to make others so. *Dr. B—y is not a stranger to it, it is known, he is credibly informed of it.*

For

For God's sake, Sir, are we in *England* or in *Spain*? Is this the language of a Familiar, who is whispering an Inquisitor against a single Heretick, or of a Protestant Divine against a great number of Gentlemen professing the Protestant Religion, and in a Protestant Country? There indeed such an accusation might be destructive to the persons it fell upon, how innocent soever: but here, thanks be to God, and to our Religious and Civil Liberties, as no proof is brought against us, we have no more to do, but to aver our innocence, absolutely to deny the charge, and plainly to tell you, that your *Informers*, how *credible* soever you may think them, are no other than a pack of base, profligate, and impudent Lyers.

This language may seem too warm: but so detestable and groundless a defamation will justify the use of it. And I must tell your self, Sir, that tho' I can by no means think you to have been the author and inventor of so horrid a calumny; yet you can never be acquitted of extreme credulity in giving ear and belief to it; nor of an unpardonable injustice in publishing and propagating it, as far as in you lay, through the nation. Surely, if you had not a very
strong

strong inclination to think and speak ill of Mathematicians, you could not have done either the one or the other. For is it at all likely or probable, that a number of persons, great part of whose employment has been to study Geometry, an excellent Logic, as you observe, * *where the definitions are clear, where the Postulata cannot be refused, nor the Axioms denied; where from the distinct contemplation and comparison of figures, their properties are derived, by a perpetual well-connected chain of consequences, the objects being still kept in view, and the attention ever fixed upon them; whereby there is acquired an habit of reasoning, close and exact and methodical: which habit strengthens and sharpens the mind, and being transferred to other objects, is of general use in the inquiring after truth; is it probable, or even possible, I would say, that a great number of persons, who have acquired this habit of reasoning, should generally not see and comprehend the clear, the certain and undeniable evidence of the Christian Religion? Credat Judæus Apella.*

Nor can I find a grain more of probability in the second article of your accusation, than in the first. Suppose a Mathematician

* *Analyst*, p. 5.

to be an Infidel, and to endeavour to make converts to Infidelity, will a deference be paid to his judgment in Divinity any more than in Law, or Physick, or any other science foreign to his own profession? Will his decisions against the Christian Religion pass even upon weak and vulgar minds, because he is thought to reason well in Geometry? *Id populus curat scilicet !* Dr. Barrow was a great and famous Mathematician; till Sir *Isaac Newton* shone out,

*Qui genus humanum ingenio superavit, & omnes
Præstrinxit, stellas exortus uti ætherius sol,*

Dr. Barrow was esteemed the greatest Geometrician in *England*: he was likewise a learned, sound and Orthodox Divine; and yet the Arrians, the Socinians, the Quakers, and the several other sects of Dissenters are still in being, though that great Master of reason both in Mathematicks and Divinity was of a contrary opinion to them all. Sir *Isaac Newton* was a greater Mathematician than any of his contemporaries in *France*; no *Frenchman* will deny it: and yet I have not heard that the French Mathematicians are converted to the Protestant Religion

by his authority, though Sir *Isaac* was known to be a zealous Protestant. When this great man had any illness, he did not trust to his own judgment for the cure of it, though so *great a master of reason*, but sent for Dr. *Mead*, and submitted himself intirely to his directions; and I suppose, upon occasion, would have paid the same deference to his Lawyer. But I am ashamed to insist so long upon a thing so evident, and shall therefore now come to that article of your charge, which you seem to have much more at heart than the interest of the Christian Religion, namely that of the errors and false reasoning of Mathematicians in their own science, or rather in that one part of their science, which is commonly called the method of Fluxions.

Your objections against this method may, I think, all of them be reduced under these three heads.

1. Obscurity of this doctrine.
2. False reasoning used in it by Sir *Isaac Newton*, and implicitly received by his followers.
3. Artifices and fallacies used by Sir *Isaac Newton*, to make this false reasoning pass upon his followers. I shall treat of these in their order.

1. Ob-

1. Obscurity of the doctrine of Fluxions.

It must be owned that this doctrine, as it is one of the most profound parts of Geometry, and perhaps the greatest instance that has ever been given of the extent of human abilities, is not without its difficulties, and will doubtless seem obscure to every unqualified or inattentive reader of Sir *Isaac Newton*. But if a person duly furnished with the necessary previous knowledge of Geometry, shall peruse his writings with that care and attention which the dignity and importance of the subject deserves ; I do assure you, Sir, from my own experience, and that of many others whom I could name, that the doctrine may be clearly conceived and distinctly comprehended. If *your imagination is strained and puzzled with it*, if it appears to you to contain *obscure and inconceivable mysteries*, in short, if you do not understand it, I tell you others do ; and you may do so too, if you will read it with due attention, and with a desire of comprehending it, rather than an inclination to censure it.

You will say, perhaps, you have already done this, and find *the clear conception of it to be impossible, and appeal to the trial of eve-*

ry *thinking reader*. Pray, Sir, who are these *thinking readers* you appeal to? Are they Geometricians, or persons wholly ignorant of Geometry? If the former, I leave it to them: if the latter, I ask, how well are they qualified to judge of the method of Fluxions? I, you will say, have represented that method to them, and from what I have laid before them, they may easily judge of it. But have you fairly, and truly, and fully represented it? Have not you very much abridged what Sir *Isaac Newton*, for the more easy and clear comprehension of his doctrine, has delivered more at large in those * parts of his works from which your account of Fluxions is taken? Have you given any account of another † part of his writings, where the foundation of this method is geometrically demonstrated, and largely explained, and difficulties and objections against it are clearly solved? Have you not altered his expressions in such a manner, as to mislead and confound your readers, instead of informing them? Where do you find Sir *Isaac Newton* using such expressions as *the velocities of the velocities, the*

* *Introd. ad Quadr. Curv. & Princ. Phil. Math. lib. 2. lemm. 2.* † *Princ. Phil. Math. lib. 1. Sect. 1.*

second,

second, third and fourth velocities, the incipient celerity of an incipient celerity, the nascent augment of a nascent augment? Is this the true and genuine meaning of the words, *fluxionum mutationes magis aut minus celeres*? Believe me, Sir, it is very easy by such pious arts as these, to make any doctrine appear absurd even to a *thinking reader*; unless instead of trusting to the representation of an adversary, he will take the pains to consult the Author that is censured. This therefore I shall advise both your *thinking reader* and your self to do, and shall betake myself to the consideration of the next head of your objections.

2. False reasoning used in the method of Fluxions by Sir *Isaac Newton*, and implicitly received by his followers.

Of this you produce two instances, both relating to a point which you justly call fundamental, namely the rule for finding the Fluxion of the rectangle of two flowing quantities; one of which instances is taken from the *Principia Philos.* and the other from the book of Quadratures.

† The objection you make to the first of these is, that the Author, in order to

† *Analyst.* p. 16, 17.

find the moment or increment of the rectangle AB , does not take the sides $A + a$, and $B + b$, as increased by their whole moments, a and b , which you say is *the direct and true method*; but instead thereof uses *the illegitimate and indirect method* of supposing the sides A and B to be *deficient or lesser* by *one half of their moments*, as $A - \frac{1}{2}a$, and $B - \frac{1}{2}b$, and then finding what the increment of the rectangle will be, when those sides come to be *increased by the other two halves of their moments*, as $A + \frac{1}{2}a$, and $B + \frac{1}{2}b$.

By the former method the increment of the rectangle AB would come out $aB + bA + ab$: by the latter it is $aB + bA$; and the difference between these two is the rectangle ab . Here you are pleased exceedingly to insult and triumph over the great Author of this method and all his followers, as being very much at a loss how to get rid of this same rectangle ab . Why, Sir, suppose they cannot in geometrical rigour get rid of it, but that it must continue to be a part of the increment of the rectangle AB ; and yet Mathematicians reject it, and instead of the rigorous geometrical increment $aB + bA + ab$, they use only $aB + bA$:
 pray,

pray, what is it you would infer from this? Will it follow that they do not *proceed scientifically*, that they are *at a loss how to conduct themselves*, that they *don't see their way distinctly*, that they *proceed blindfold*? Will you say that they do not exactly see the consequence of this omission, through their whole proceeding, and do not certainly and clearly know how far their conclusion will be affected by it? Do not they know that in estimating any finite quantity how great soever, proposed to be found by the method of Fluxions, a globe, suppose, as big as that of the earth, or, if you please, of the sun, or of the whole planetary system, or even the orb of the fixed stars; do not they know, I say, and are they not able clearly and invincibly to demonstrate that, in so immense a magnitude, this omission shall not cause them to deviate from the truth so much as a single pin's head, nay not the thousandth, not the millionth part of a pin's head? How then can it be said that *they proceed without clearness and without science*, and don't know what they are doing, nor whither this method is carrying them?

I shall here beg leave, for the sake of
readers

readers less mathematically qualified, to put a very easy and familiar case. Suppose two Arithmeticians to be disputing whether vulgar fractions are to be preferred to decimal; would it be fair in him who is for expressing the third part of a farthing by the vulgar fraction $\frac{1}{3}$, to affirm that his antagonist proceeded blindfold, and without knowing what he did, when he pretended to express it by 0.33333 &c. because this expression did not give the rigorous, exact value of one third of a farthing? Might not the other reply that, if this expression was not rigorously exact, yet it could not be said he *proceeded blindfold, or without clearness and without science* in using it, because by adding more figures he could approach as near as he pleased, and wherever he thought fit to stop, he could clearly and distinctly find and demonstrate how much he fell short of the rigorous and exact value? Might not he further say that as the &c. implied all the possible repetitions of the figure 3, even to infinity, therefore his expression did not differ by any the least assignable quantity from the other value, $\frac{1}{3}$; and that as he knew and clearly conceived

that it did so, he could not justly be said to be in any error, much less to act in the dark, when he used that expression?

Having observed that this omission, or error as you are pleased to call it, in rejecting the rectangle $a b$, is at most such an one as can cause no assignable difference, how small soever, in the conclusions drawn from the method of Fluxions; and that Mathematicians in committing this error, do nevertheless proceed scientifically and with their eyes open, as having a clear and distinct view not only of the original error, but of the effect of it both in every step they take, and in all the conclusions they draw from this method; I shall now for your farther satisfaction proceed to examine, whether in reality it be any error at all; and whether $a B + b A$ be not in strict, geometrical rigour, the true fluxion, moment, or increment of the rectangle $A B$. And as you say *the foreign Mathematicians are supposed by some, even of our own, to proceed in a manner, less accurate perhaps and geometrical, yet more intelligible, than that of Sir Isaac Newton*; I shall first consider what course they take in rejecting the rectangle $a b$, and

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shall

shall then proceed to Sir *Isaac Newton's* way of excluding it.

The famous *Marquis de l'Hospital*, whom I the rather follow, because he is thought to have written upon this subject with greater perspicuity than any other foreign Mathematician, as also because you expressly quote him, after he has found the fluxion of the rectangle * $x y$, composed of the two variable quantities x and y , to be $y d x + x d y + d x d y$; does afterwards reject the rectangle $d x d y$, and thereby leave only $y d x + x d y$ for the fluxion of $x y$. Upon which you observe, † *as to the method of getting rid of this quantity $d x d y$, equivalent to the rectangle $a b$ of Sir Isaac Newton, that it is done without the least ceremony.* Is this true? Does not the Marquis, in this very proposition ‡ quoted by you, and at the very instant that he rejects that quantity, give this reason for it, that it is infinitely small with respect to the other terms $x d y$ and $y d x$? Does he not immediately after give a demonstration of its being so? And does he not, to shew that he has a right to

* *Analyse des infiniment petits. Prop. 2.*

† *Analyst. p. 28.*

‡ *ibid. p. 26.*

reject

reject it upon that reason, plainly and expressly refer his reader to his first *postulatum* or supposition? Does he not in that *postulatum* expressly require it should be allowed him, *that a quantity, which is augmented or diminished by another quantity infinitely less than the first, may be considered as if it continued the same, i. e. had received no such augmentation or diminution?* Is not this plainly the case of the quantity $x \, d \, y + y \, d \, x$, augmented or diminished by the quantity $d \, x \, d \, y$? Have you then done fairly and justly by this Great Man, in concealing all this from your *thinking readers* to whose judgment you refer your self, and in telling them *that this quantity is rejected without the least ceremony?*

You will tell me perhaps, that you do not allow of this *postulatum*. Why then you must not read the *Marquis de l'Hospital's* book. The *postulatum* is placed at the very beginning of it, as an express declaration to his readers, that unless that be allowed him, he will not undertake to demonstrate what follows. If you admit his *postulatum*, you will find him proceed clearly and evidently and like a Mathematician in rejecting the quantity $d \, x \, d \, y$: if not, you

have no right to attack his proposition founded upon that *postulatum*, but only to give your reasons against the *postulatum* itself. And thus much in vindication of my first master, that great and clear-headed Geometrician the *Marquis de l'Hospital*, whose only misfortune it was to have met with muddy waters, and not to have drunk of the fountain itself.

I come now to consider what course Sir *Isaac Newton* himself has taken to avoid this formidable rectangle *a b*, this fatal rock, this *Bishop and his Clerks*, that threatens destruction to him and all his followers. And here, Sir, in order to give your reasoning its full force, I shall transcribe the greatest part of the ninth section of your discourse, after which I shall do the same justice to Sir *Isaac Newton*, by giving his demonstration in his own words. Your ninth section begins thus.

“ Having considered the object, I proceed to consider the principle of this
 “ new Analysis by Momentums, Fluxions,
 “ or Infinitesimals; wherein if it shall appear that your capital points, upon which
 “ the rest are supposed to depend, include
 “ error and false reasoning; it will then
 “ follow

“ follow that you, who are at a loss to con-
 “ duct your selves, cannot with any decen-
 “ cy set up for guides to other men. The
 “ main point in the method of Fluxions is
 “ to obtain the fluxion or momentum of
 “ the rectangle or product of two indeter-
 “ minate quantities. Inasmuch as from
 “ thence are derived rules for obtaining
 “ the Fluxions of all other products and
 “ powers; be the coefficients or the indexes
 “ what they will, integers or fractions, ra-
 “ tional or surd. Now this fundamental
 “ point one would think should be very
 “ clearly made out, considering how much
 “ is built upon it, and that its influence
 “ extends throughout the whole Analysis.
 “ But let the reader judge. This is given
 “ for demonstration. * Suppose the product
 “ or rectangle AB increased by continual
 “ motion: and that the momentaneous in-
 “ crements of the sides A and B are a and
 “ b . When the sides A and B were defi-
 “ cient, or lesser by one half of their mo-
 “ ments, the rectangle was $\overline{A - \frac{1}{2}a} \times$
 “ $\overline{B - \frac{1}{2}b}$, i. e. $AB - \frac{1}{2}aB - \frac{1}{2}bA +$
 “ $\frac{1}{4}ab$. And as soon as the sides A and B
 “ are increased by the other two halves of

* *Philos. Nat. Princ. Math. Lib. II. Lemm. 2.*

“ their

“ their moments, the rectangle becomes
 “ $A + \frac{1}{2}a \times B + \frac{1}{2}b$, or $AB + \frac{1}{2}aB$
 “ $+ \frac{1}{2}bA + \frac{1}{2}ab$. From the latter rect-
 “ angle subduct the former, and the remain-
 “ ing difference will be $aB + bA$. There-
 “ fore the increment of the rectangle gene-
 “ rated by the intire increments a and b
 “ is $aB + bA$. Q. E. D. But it is plain
 “ that the direct and true method to obtain
 “ the moment or increment of the rectangle
 “ AB , is to take the sides as increased by
 “ their whole increments, and so multiply
 “ them together, $A + a$ by $B + b$, the
 “ product whereof $AB + aB + bA +$
 “ ab is the augmented rectangle; whence
 “ if we subduct AB , the remainder aB
 “ $+ bA + ab$ will be the true increment
 “ of the rectangle, exceeding that which
 “ was obtained by the former illegitimate
 “ and indirect method by the quantity
 “ ab . And this holds universally be the
 “ quantities a and b what they will, big or
 “ little, finite or infinitesimal, increments,
 “ moments, or velocities. Nor will it avail
 “ to say that ab is a quantity exceeding
 “ small: Since we are told * that *in rebus*
 “ *mathematicis errores quam minimi non sunt*

* *Introduct. ad Quadrat. Curv.*

“ *contemnendi*. Such reasoning as this, for
 “ demonstration, nothing but the obscu-
 “ rity of the subject could have encouraged
 “ or induced the great author of the fluxion-
 “ ary method to put upon his followers,
 “ and nothing but an implicate deference
 “ to authority could move them to
 “ admit. The case indeed is difficult.
 “ There can be nothing done till you have
 “ got rid of the quantity $a b$. In order to
 “ this the notion of Fluxions is shifted :
 “ It is placed in various lights : Points which
 “ should be as clear as first Principles are
 “ puzzled ; and terms which should be stea-
 “ dily used are ambiguous. But notwith-
 “ standing all this address and skill the point
 “ of getting rid of $a b$ cannot be obtained
 “ by legitimate reasoning.

It is now time to hear Sir *Isaac Newton*.

Princip. Lib. II. Lemm. 2. Cas. 1. “ Rec-
 “ tangulum quodvis motu perpetuo auctum
 “ $A B$, ubi de lateribus A & B deerant mo-
 “ mentorum dimidia $\frac{1}{2} a$ and $\frac{1}{2} b$, fuit $A -$
 “ $\frac{1}{2} a$ in $B - \frac{1}{2} b$, seu $A B - \frac{1}{2} a B - \frac{1}{2} b$
 “ $A + \frac{1}{2} a b$; & quamprimum latera A &
 “ B alteris momentorum dimidiis aucta sunt,
 “ evadit $A + \frac{1}{2} a$ in $B + \frac{1}{2} b$, seu $A B + \frac{1}{2}$
 “ $a B$

“ $a B + \frac{1}{2} b A + \frac{1}{2} a b$. De hoc rectan-
 “ gulo subducatur rectangulum prius, &
 “ manebit excessus $a B + b A$. Igitur late-
 “ rum incrementis totis a & b generatur rec-
 “ tanguli incrementum $a B + b A$. Q.E.D.

Having now fairly laid before my reader what both your self and Sir *Isaac Newton* have delivered upon this subject, I come to examine which of you is in the right.

In the first place, I find you take it for granted that what Sir *Isaac Newton* is here endeavouring to find, by supposing the sides A and B first to want one half of their moments, and afterwards to have gained the other halves of their moments, is the increment of the rectangle AB . In this I conceive you are mistaken. For neither in the demonstration itself, nor in any thing preceding or following it, is any mention so much as once made of the increment of the rectangle AB . On the contrary it plainly appears that what he endeavours to obtain by these suppositions, is no other than the increment of the rectangle $\overline{A - \frac{1}{2} a} \times \overline{B - \frac{1}{2} b}$, and you must own that he takes *the direct and true method to obtain it*. But you will say, is it not the business of this lemma to determine the moments of flow-
 4 ing

ing quantities? And is it not the design of Case 1 to determine the moment of the rectangle AB ? I answer that it is so: but that rigorously speaking the moment of the rectangle AB , is not, as you suppose, the increment of the rectangle AB ; but it is the increment of the rectangle $A - \frac{1}{2}a \times B - \frac{1}{2}b$. In order to clear up this point, I must observe,

1. That the word moment is used by Sir *Isaac Newton* and your self to signify indifferently either an increment, or a decrement.

2. That $aB + bA + ab$ is by you demonstrated to be the true increment of the rectangle AB .

3. That $aB + bA - ab$ is the true decrement of the same rectangle AB ; as plainly appears upon taking the same *true and direct method* for finding the decrement; as you have used for finding the increment.

Now, Sir, I would humbly beg leave to inquire of you, who see so much more clearly into these matters than Sir *Isaac Newton* or any of his followers; which of these two Quantities, $aB + bA + ab$ and $aB + bA - ab$, you will be pleased to call the moment of the rectangle AB ? The

case indeed is difficult, The difference between them is no less than $2ab$, just the double of that same ab , which has given us all so much trouble; and yet each of them plead an equal right to the title of moment. So equal a one, that, though I am very sensible of your *address and skill*, yet there seems to be no possibility of deciding the controversy between them *by legitimate reasoning*. I see but two ways of doing it. One is that they should toss up cross or pile for the title: Or if that be thought too boyish and unbeseeming the Gravity of Mathematical quantities, they must even end the dispute in an amicable manner, and without claiming any preference one of another, agree that they make two moments between them. Then, Sir, I apprehend the case will stand thus: $aB + bA + ab + aB + bA - ab$ making twice the moment of the rectangle AB ; it follows that $aB + bA$ will make the single moment of the same rectangle.

You see, Sir, after all the pains you have taken, this affair comes out, even upon your own concessions, just as Sir *Isaac Newton* and his followers would have it. Believe me, there is no remedy. You must acquiesce.

Only, if it may be any Satisfaction to you to know why Sir *Isaac* took this indirect way of finding the increment of $A - \frac{1}{2} a \times B - \frac{1}{2} b$, instead of proceeding directly to find the moment of the rectangle $A B$, I shall be ready to oblige you as far as can be expected from one of those, who *have shown themselves more eager in applying his method, than accurate in examining his principles.*

The final cause or motive to this proceeding, I find, is not unknown to you; you say it is *very obvious*, meaning, I suppose, that thereby it was intended to exclude this same troublesome rectangle $a b$. Why truly, Sir, in a book of strict demonstration, as Sir *Isaac Newton* intended his *Principia* should be, it was certainly more proper to exclude that quantity, so as not to suffer it to appear, than first to introduce it into the reader's view and then to reject it.

You add that *it is not so obvious or easy to explain a just and legitimate reason for it, or shew it to be Geometrical.* How far it may be obvious or easy to assign such a reason, I will not dispute: though I am apt to think that what is easy to me, cannot be difficult to other persons, provided they

use the same endeavours to find the truth, as I have done. Now, I apprehend the reason of this proceeding of Sir *Isaac Newton* to be the following very plain one: That in order to find the moment of the rectangle AB , it is more consonant to strict Geometrical rigour to take the increment of the rectangle $\overline{A - \frac{1}{2}a} \times \overline{B - \frac{1}{2}b}$, than to take the increment of the rectangle AB itself. And if I can make this appear, you must allow that he had a *just and legitimate reason for proceeding* as he did.

You know very well that the moment of the rectangle AB is proportional to the velocity of that rectangle, with which it alters, either in increasing, or in diminishing. Now, I ask, in Geometrical rigour what is properly the velocity of this rectangle? Is it the velocity with which the rectangle from AB becomes $\overline{A + a} \times \overline{B + b}$; or the velocity with which from AB it becomes $\overline{A - a} \times \overline{B - b}$? I find my self exactly in the case of the Afs between the two bottles of hay: I see no reason, nor possibility of a reason to determine me either one way, or the other. But methinks I hear the venerable Ghost of Sir *Isaac Newton* whisper

whisper me, that the velocity I seek for,
 is neither the one nor the other of these,
 but is the velocity which the flowing rect-
 angle has, not while it is greater or less
 than AB , neither before, nor after it be-
 comes AB , but at that very instant of time
 that it is AB . In like manner the mo-
 ment of this rectangle is neither the in-
 crement from AB to $\overline{A+a} \times \overline{B+b}$;
 nor is it the decrement from AB to
 $\overline{A-a} \times \overline{B-b}$: It is not a moment com-
 mon to AB and $\overline{A+a} \times \overline{B+b}$, which
 may be considered as the increment of the
 former, or as the decrement of the latter:
 Nor is it a moment common to AB and
 $\overline{A-a} \times \overline{B-b}$, which may be considered
 as the decrement of the first, or as the
 increment of the last: But it is the
 moment of the very individual rect-
 angle AB itself, and peculiar to that on-
 ly; and such as being considered indiffe-
 rently either as an increment or decrement,
 shall be exactly and perfectly the same. And
 the way to obtain such a moment is not
 to look for one lying between AB and
 $\overline{A+a} \times \overline{B+b}$; nor to look for one ly-
 ing between AB and $\overline{A-a} \times \overline{B-b}$:
 that

that is, not to suppose AB as lying at either extremity of the moment; but as extended to the middle of it; as having acquired the one half of the moment, and as being about to acquire the other; or as having lost one half of it, and being about to lose the other. And this is the method Sir *Isaac Newton* has taken in the demonstration you except against.

What say you, Sir? Is this a *just and legitimate reason* for Sir *Isaac's* proceeding as he did? I think you must acknowledge it to be so. For even if you should still have any doubt whether his proceeding be rigorously Geometrical; yet you cannot but confess that whether moments be considered as infinitely small, or as finite quantities, his method approaches nearer to Geometrick rigour, than that which you propose. I think likewise you cannot but be sensible of great want of caution in your own proceeding; inasmuch as that quantity, which Sir *Isaac Newton* through this whole Lemma, and all the several cases of it, constantly calls a moment, without confining it to be either increment or decrement, is by you inconsiderately, and arbitrarily, and without any shadow of reason given, supposed and determined

terminated to be an increment. And this, Sir,
 naturally leads me to give you a piece of
 friendly advice, which you seem to stand
 much in need of. It is that, whenever you
 take it into your head to criticise upon Sir
Isaac Newton's writings, you first examine
 and weigh every word he uses; and if you
 translate him, keep closely to his expression.
 Believe me, this Great Man, among his o-
 ther extraordinary indowments, had a pecu-
 liar sagacity in foreseeing objections, as well
 as an aversion to disputing. To these two
 qualities accompanied with extreme huma-
 nity and condescension it is owing, that he
 uses such accuracy in his expression, that
 an intelligent and attentive reader can ne-
 ver mistake him; and that he does of him-
 self first propose, and then remove such
 difficulties, as may naturally arise in the
 minds of even candid and judicious persons,
 who are not yet masters of the subject he
 treats of. But as for the *Homines stolidi &*
ad depugnandum parati, he contents himself
 with observing that prudent caution in eve-
 ry word he uses, that as they shall find no-
 thing to mislead them, so on the other
 hand, if they undeservedly and unadvisedly
 attack him, they shall certainly and una-
 voidably

voidably *induere se in stimulos latentes*, and expose themselves to the scorn and contempt of every unprejudiced observer.

This great example, which in any the lowest degree to imitate is the highest honour I can ever arrive at, or even desire, moves me to propose and remove an objection which may possibly arise in your mind, and hinder you from acquiescing in one part of what I have just now laid before you. It is that I have supposed the rectangle AB extended to the middle of its moment; as having acquired the one half of it, and being about to acquire the other; or as having lost one half of it, and being about to lose the other. You may say this is strictly and exactly true in respect of the sides of that rectangle; which sides, from $A - \frac{1}{2}a$ and $B - \frac{1}{2}b$, are become A and B ; and are about to become $A + \frac{1}{2}a$ and $B + \frac{1}{2}b$: but that it is not equally true of the rectangle composed of those sides, which from $\overline{A - \frac{1}{2}a} \times \overline{B - \frac{1}{2}b}$, or $\overline{AB - \frac{1}{2}aB - \frac{1}{2}bA + \frac{1}{4}ab}$, is become $\overline{AB}$; and is about to become $\overline{A + \frac{1}{2}a} \times \overline{B + \frac{1}{2}b}$, or $\overline{AB + \frac{1}{2}aB + \frac{1}{2}bA + \frac{1}{4}ab}$: since the part of the moment which AB is supposed to have gained

gained, namely $\frac{1}{2} a B + \frac{1}{2} b A - \frac{1}{4} a b$, is not equal to that part of the moment which is about to be gained, namely $\frac{1}{2} a B + \frac{1}{2} b A + \frac{1}{4} a b$; the difference between them being $\frac{1}{2} a b$. In answer to this I reply, that these two quantities, $\frac{1}{2} a B + \frac{1}{2} b A - \frac{1}{4} a b$, and $\frac{1}{2} a B + \frac{1}{2} b A + \frac{1}{4} a b$, so long as a and b are finite quantities, are undoubtedly unequal; but that the more a and b are diminished, by so much nearer will these quantities approach to an equality; and if a and b are diminished *ad infinitum*, the two quantities will then be perfectly equal. See this demonstrated *Princip. Lib. I. Sect. 1. Lemm. 1.* Which *Lemma*, for your own sake and mine, I could wish you had consulted sooner.

Lastly, to remove all scruple and difficulty about this affair, I must observe, that the moment of the rectangle AB , determined by Sir *Isaac Newton*, namely $a B + b A$, and the increment of the same rectangle, determined by yourself, namely $a B + b A + a b$, are perfectly and exactly equal, supposing a and b to be diminished *ad infinitum*; and this by the *Lemma* just now quoted.

I now come to your second instance of false reasoning, which you take from the

Book of Quadratures; and passing by the *Lemma* you so gravely lay down to shew, that when two contrary suppositions are made, nothing can be inferred from either of them; as a truth that no School-boy can be ignorant of; I shall here transcribe this instance of false reasoning as you give it, with your observations upon it.

* “ Let the quantity x flow uniformly,
“ and be it proposed to find the Fluxion of

“ x . In the same time that x by flowing

“ becomes $x + o$, the Power x^n becomes

“ $\overline{x + o}^n$, i. e. by the method of infinite Se-

“ ries $x^n + n o x^{n-1} + \frac{n n-1}{2} o o x^{n-2} + \&c.$

“ and the increments o and $n o x^{n-1} + \frac{n n-1}{2}$

“ $o o x^{n-2} + \&c.$ are one to another as 1 to

“ $n x^{n-1} + \frac{n n-1}{2} o x^{n-2} + \&c.$ Let now

“ the increments vanish, and their last pro-

“ portion will be 1 to $n x^{n-1}$. But it should

“ seem that this reasoning is not fair or con-

“ clusive. For when it is said, let the incre-

* *Analyst* p. 20.

“ ments

“ ments vanish, i. e. let the increments be
 “ nothing, or let there be no increments, the
 “ former supposition that the increments
 “ were something, or that there were incre-
 “ ments, is destroyed, and yet a consequence
 “ of that supposition, i. e. an expression
 “ got by virtue thereof, is retained. Which,
 “ by the foregoing *Lemma*, is a false way
 “ of arguing. Certainly when we suppose
 “ the increments to vanish, we must sup-
 “ pose their proportions, their expressions,
 “ and every thing else derived from the
 “ supposition of their existence to vanish
 “ with them.

You are pleased to go on for some num-
 ber of pages, *to make this point plainer, to un-
 fold the reasoning, and to propose it in a fuller
 light.* But I think we may as well stop
 here. You have already so fully *unfolded*
 it, that if this be the way of reasoning of
 our Mathematical Infidels, I pronounce our
 Religion out of all danger from that quar-
 ter. From this time our Reverend Clergy
 may sleep in quiet, and be under as little
 apprehension from the unbelieving Analyst,
 as from the most ignorant of the Popish
 Monks, the most stupid of the Jewish Rab-
 bi's, or the most empty and contemptible

praters among the Minute Philosophers. I have only one doubt upon me. Pray, Sir, are you very sure that this is the real doctrine of Sir *Isaac Newton*? Are you absolutely certain you have not mistaken him? You seem, I must confess, to be exceedingly cautious, you blame others for not being *accurate in examining his Principles*, you talk of *preventing all possibility of mistaking you*, and you treat him and his followers in such a manner, that you are to expect no quarter from them in case of ill success. And yet this is so great, so unaccountable, so horrid, so truly Bœotian a blunder, that I know not how to think a *Great Genius*, a *Newton* could be guilty of it. For God's sake let us examine it once more. *Evanescent jam augmenta illa, let now the increments vanish, i. e. let the increments be nothing, or let there be no increments.* Hold, Sir, I doubt we are not right here. I remember Sir *Isaac Newton* often uses the terms of *momenta nascentia* and *momenta evanescentia*. I think I have seen you likewise several times using the like terms of nascent and evanescent increments. Also, if I am not mistaken, both he and you consider a nascent, or evanescent moment, an increment or decrement,

crement, as the same quantity under different circumstances; sometimes as in the point of beginning to exist, and other times as in the point of ceasing to exist. From this methinks it should follow that the two expressions subjoined, will be perfectly equivalent to each other.

Nascantur jam augmenta illa, & eorum ratio prima erit

Evanescent jam augmenta illa & eorum ratio ultima erit

The meaning of the first can possibly be no other than to consider the first proportion between the nascent augments, in the point of their beginning to exist. Must not therefore the meaning of the latter be to consider the last proportion between the evanescent augments, in the point of evanescence, or of their ceasing to exist? Ought it not to be thus translated, Let the augments now become evanescent, let them be upon the point of evanescence? What then must we think of your interpretation, *Let the increments be nothing, let there be no increments?* Do not the words *ratio ultima* stare us in the face, and plainly tell us that though there

there is a last proportion of evanescent increments, yet there can be no proportion of increments which are nothing, of increments which do not exist? I believe, Sir, every thinking person will acquit Sir *Isaac Newton* of the gross oversight you ascribe to him, and will acknowledge that it is your self alone, who have been guilty of a most palpable, inexcusable, and unpardonable blunder. I now come to the third head of your objections.

3. Arts and fallacies used by Sir *Isaac Newton* to make his false reasoning pass upon his followers.

On this head I shall not need to take up much of your time, because having already fully proved that Sir *Isaac Newton* was not guilty of false reasoning in the instances you alledge, I suppose no body will think he had any occasion to make use of arts and fallacies to impose upon his followers. But you have one observation upon this head, which is so very singular, that I cannot but think it worthy of particular consideration. *Considering, * say you, the various arts and devices used by the Great Author of the Fluxionary method: in how many lights he placeth his Fluxions: and in what different ways he*

* *Analyst* p. 27.

attempts

attempts to demonstrate the same point: one would be inclined to think he was himself suspicious of the justness of his own demonstrations: and that he was not enough pleased with any one notion steadily to adhere to it. Thus much at least is plain, that he owned himself satisfied concerning certain points, which nevertheless he could not undertake to demonstrate to others. Really, Sir, this seems to be very hard usage. Sir Isaac Newton has made a new and great discovery, by which he has not only out-done all the Geometricians that ever went before him, but can enable such ordinary proficient in Mathematicks, as you and I, to surpass all the great masters of antiquity: He is so good as to instruct us in this method; and because it requires some pains and discernment to comprehend it rightly, he sets it in several various lights, that by means of some of these we may not fail of understanding it. Pray, Sir, have you and I any reason to complain of this? For my part, I think myself greatly obliged to him for his condescension: If he had not taken so much pains to explain his doctrine, I doubt I should never have understood it. But, for God's sake, what is it you are offended at, who do not still
under-

understand him! You are all in the dark, and yet are angry at his giving you so much light. Surely the fault is not in Sir *Isaac Newton*, but in your own eyes.

So thick a drop serene has quench'd their
orbs,
Or dim suffusion veil'd.

But is not he *himself*, say you, *suspicious of the justness of his own demonstrations?*

Pray, Sir, when a Divine is instructing his hearers in a weighty and important point of Religion, if from a desire that every one should perfectly understand him, he is at the pains to use several arguments, and to set his Doctrine in various lights; would it be reasonable, or just, or grateful in any of his auditors to infer from this, that the Preacher was suspicious of the justness of his own reasoning? When you, after all the demonstrations that had been given of the being of a God, by the learned Fathers of the Church, and by the wisest of the Philosophers of all ages, thought fit to introduce that new and singular one of a *Visual Language*, would it be fair in me to suppose that you were suspicious of all the
former

former proofs of the existence of a Deity, and left that great and important truth to depend upon a metaphorical argument? Surely one argument may be just, and conclusive, and perfectly satisfactory to him that uses it; and yet the matter treated of may be of that difficulty, or of that dignity and importance, as not only to admit of, but to require several others for the instruction and conviction of his hearers. And thus much may suffice for your third and last head of objections against Sir *Isaac Newton* and his followers: Only before I conclude I must advise you to correct one word in your extract from his * Letter to Mr. *Collins*, Nov. 8, 1676. or rather to give up that extract intirely, as being of no manner of service to you. There is a great deal of difference between saying I cannot undertake to prove a thing, and I will not undertake it. Sir *Isaac*, in that Letter says, I will not: And besides, the point there mentioned is not the point here in debate; so that you have no right to draw any inference from that point to this.

Having now done with every thing necessary to the vindication of Sir *Isaac Newton* and his followers, and thereby driven

* *Analyst*. p. 27.

you entirely out of our intrenchments, I am considering whether I should sally out and attack you in your own. You have thrown up some works, I see, which at a distance make a pretty good appearance, and seem capable of defence: But upon taking a nearer view of them, I judge them to be very slight and untenable, and to be guarded rather by a new-raised, undisciplined Militia, than any thing of veteran, regular Troops; so that it would not be very difficult to carry them by assault. But as they seem rather designed for shew, than use, more to amuse yourself, than any way to annoy us, I am determined to leave you in possession of them.

Only your supposition of a * double error in the method of Fluxions, and the use you make of it to shew how true conclusions are obtained from false principles, by means of two contrary mistakes exactly compensating one another, has something in it so extraordinary, as to require and deserve a particular consideration. This darling Phantom, this beloved offspring of your teeming brain, which like *Minerva* issuing armed from the head of *Jupiter*, her spear

* *Analyst* p. 31 & seq. to p. 49.

in one hand, and her Shield with the Gorgon's head in the other, is to turn all our Mathematicians into stocks, and stones, and statues, is set forth with so much art and skill, and is dressed out in so advantageous and pompous a manner, to draw the attention and to dazzle the imagination of the spectators, that it would be unpardonable neglect and rudeness in me to pass it by unregarded. I shall not therefore content my self with saying that one * of these errors is already become evanescent, i. e. *is nothing, is no error at all*; and that the other of them will likewise immediately disappear like † *the Ghost of a departed quantity*, if you exorcise it with a few words out of the first section of the *Principia*: On the contrary, I propose so far to gratify your fondness for this hopeful scheme, as to give it a fair and full examination.

We are to consider therefore what may be the reason, that in the method of Fluxions the conclusions are exactly true: For in the exactness of the conclusions we are both agreed; though there be a wide difference between us in respect of the means by which Mathematicians arrive at that

* See p. 46—53.

† *Anal.* p. 59.

exactness. I conceive that the conclusion is therefore exact, because it is deduced by just reasoning from certain principles. You on the contrary are of opinion that Sir *Isaac Newton* is guilty of a capital and fundamental error in rejecting the quantity $a b$, so often talked of, and * *that the conclusion comes out right, not because the quantity rejected is infinitely small; but because this error is compensated by another contrary and equal error.* And this you say, † *perhaps the Demonstrator himself never knew or thought of.* ‡ *If he had committed only one error, he would not have come at a true solution of the Problem. But by virtue of a two-fold mistake he arrives, though not at science, yet at truth. For science it cannot be called, when he proceeds blind-fold, and arrives at the truth not knowing how or by what means.* This is the way you account for what you justly say, *may perhaps seem an unaccountable Paradox,* § *that Mathematicians should deduce true Propositions from false Principles, be right in the conclusion, and yet err in the premisses; that error should bring forth truth, though it cannot bring forth science.*

Now truly, Sir, if this Paradox of yours

* *Anal.* p. 35. † p. 36. ‡ p. 34. § p. 31.
should

should be well made out, I must confess it
 ought very much to alter the opinion the
 world has had of Sir *Isaac Newton*, and oc-
 casion our talking of him in a very different
 manner from what we have hitherto done.
 What think you if, instead of the greatest
 that ever was, we should call him the most
 fortunate, the most lucky Mathematician
 that ever drew a circle? Methinks I see the
 good old Gentleman fast asleep and snoring
 in his easy chair, while Dame Fortune is
 bringing him her apron full of beautiful The-
 orems and Problems *which he never knows*
or thinks of: just as the *Athenians* once painted
 her dragging towns and cities to her favourite
 General. For what else but extreme good
 fortune could occasion the conclusions arising
 from his method to be always true and just
 and accurate, when the premisses were in-
 accurate and erroneous and false, and only
 led to right conclusions by means of two
 errors ever compensating one another to the
 utmost exactness? What luck was here?
 That when he had made one capital, funda-
 mental, general mistake, he should happen
 to make a second, as capital, as fundamen-
 tal, as general as the first; That he should
 not proceed to commit three or four such
 mistakes,

mistakes, but stop at the second : That these two mistakes should chance not to lie both the same way, but on contrary sides, so that the one might help to correct the other; and lastly, that the two contrary errors, among all the infinite proportions which they might bear to one another, should happen upon that of a perfect equality; so that one might in all possible cases be exactly balanced or compensated by the other. With a quarter of this good fortune a man might get the 10000 /. prize in the present Lottery, with a single Ticket.

But to come to our point, we are to examine whether the exactness of the conclusion is owing to the exact compensation of of one of these errors by the other, or to those errors being utterly insignificant, being in reality no errors at all. And in order thereto I propose to see how the conclusion will come out, when only one of these errors is committed, so that there is nothing to compensate it.

In your 21 Section, which with its figure I here refer to, the first error is supposed to be the making the subtangent or $S = \frac{RB \times PB}{RN}$, instead of $S = \frac{RB \times PB}{RL}$

The

The second error is making $dy = \frac{p dx}{2y}$,
 instead of $dy = \frac{p dx - dy dy}{2y}$.

If both these errors be committed, or if neither of them be committed, the conclusion is agreed to be equally just and right, giving $S = 2x$.

If I avoid the first of these errors, by making $S = \frac{RB \times PB}{L}$; and retain the second, by supposing $dy = \frac{p dx}{2y}$; my conclusion will be $S = 2x \times \frac{2y}{2y + dy}$.

On the other hand if I commit the first error, and avoid the second, my conclusion will give me $S = 2x \times \frac{2y + dy}{2y}$.

Now I affirm that these two several values of S , which are the result of one error only without any thing to compensate it, are both true and equally exact with the former value, $2x$, which is the result either of two errors, or of none at all. You, I am sensible, will dispute this with me; you will say that one of these, $2x \times \frac{2y}{2y + dy}$, is less than $2x$;
 and

and the latter, $2x \times \frac{2y + dy}{2y}$, is in the same proportion bigger than $2x$. I beg leave therefore, for the information of some of my readers, to ask you a question. Supposing the true subtangent $2x$ to be a thousand miles in length, how much will the second value of that subtangent, $2x \times \frac{2y}{2y + dy}$ fall short of a thousand miles? Will it be a yard, or a foot, or an inch? None of these you confess, nor the thousandth, nor the thousand-millionth part of an inch.

I ask farther, what then is this difference? Is it possible in all the infinity of fractional numbers to find any thing small enough to represent it? You own, you confess it is not: You must confess likewise, that if these three several values of S were all to be expressed in numbers, without being reducible to which, in your * opinion, they can be of no use, they must every one be expressed by 1000, without the least title of variation, addition, or diminution. Behold, Gentle Reader, what a mighty † beam here has been discovered in the eyes of Mathematicians, in comparison of which

* *Analyst Query 24.*

† *Motto to Analyst.*

all the difficulties in Divinity are but as motes and atoms!

Since therefore these errors are wholly insignificant, my conclusion when reduced to numbers, coming out exactly the same, whether the first, or second, or neither, or both of these errors be committed; and since by committing both these errors, the calculus, which would otherwise, especially in the higher operations, be exceedingly tedious and laborious, is now rendered surprisingly expeditious and easy; it seems to me that this is so far from being any defect in the method of Fluxions, that on the contrary it is one of the greatest advantages and excellencies of that invention. But you tell me it is not the usefulness of this method that is the matter in dispute: all the question is whether it be scientific, whether those who use it, *see their way distinctly, or proceed blindfold and arrive at the truth not knowing how or by what means.* I have spoken to this before, but must add a word or two more in this place. You, Sir, are for avoiding these two errors; I am for retaining them. When you avoid them, do not you *see your way distinctly*? And if I retain them, voluntarily, and with my eyes open;

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may

may I not nevertheless clearly see the effect of these errors, or of either of them, in every step I take and in the conclusion I at last come to? May I not therefore likewise be said to *see my way distinctly*? Now, if you and I can see our way so well, I am afraid it will be construed as great presumption in us to suppose that no body does so besides our selves: and much more, if we should say that the Great Inventor of this method, and the Author of so many other wonderful discoveries, *never knew or thought of* what to us appears so plain and manifest; that he who gave us so much light, was in the dark himself; that he who opened our Eyes, had no sight of his own. For my part I can never concur with you in thinking that I see farther, or go beyond Sir *Isaac Newton*:

Sed longe sequor, & vestigia pronus adoro.

But if you think fit to persist in asserting that this affair of a double error is intirely a new discovery of your own, which Sir *Isaac* and his followers *never knew nor thought of*, I have unquestionable evidence to convince you of the contrary. I must acquaint you therefore with what all his followers

lowers are already apprised of, that these very objections of yours were long since foreseen, and clearly and fully removed by Sir *Isaac Newton*, in the first section of the first book of his *Principia*; the greater part of which section, particularly the first and seventh *Lemma*, and that admirable *Scholium* at the end of it, was written to this very end and purpose only, and to no other in the world.

I have now no more to do, but only to acquit my self of the promise I made a while ago, to rectify a mistake you are fallen into with regard to another of the greatest men the English nation has produced. In order to which I must here transcribe the greater part of the CXXV article of your New Theory of Vision.

“ After reiterated endeavours to apprehend the general Idea of a Triangle, I have found it altogether incomprehensible, And surely if any one were able to introduce that Idea into my Mind, it must be the Author of the *Essay concerning Human Understanding*; He, who has so far distinguished himself from the generality of Writers, by the clearness and significance of what he says. Let us therefore see how this celebrated Author describes

“ the general, or abstract Idea of a Trian-
 “ gle. *It must be, says he, neither Oblique,*
 “ *nor Rectangular, neither Equilateral, E-*
 “ *quicrural, nor Scalenum; but all and none*
 “ *of these at once. In effect it is somewhat*
 “ *imperfect that cannot exist; an Idea, where-*
 “ *in some parts of several different and incon-*
 “ *sistent Ideas are put together.* Essay on
 “ Human Understanding. B. iv. C. 7. S. 9.
 “ This is the Idea, which he thinks need-
 “ ful for the Enlargement of Knowledge,
 “ which is the subject of Mathematical De-
 “ monstration, and without which we could
 “ never come to know any general Proposi-
 “ tion concerning Triangles. That Author
 “ acknowledges it doth *require some pains*
 “ *and skill to form this general Idea of a Tri-*
 “ *angle.* Ibid. But had he called to mind
 “ what he says in another place; to wit,
 “ that *Ideas of mixed Modes wherein any in-*
 “ *consistent Ideas are put together, cannot so*
 “ *much as exist in the mind, i. e. be con-*
 “ *ceived.* Vid. B. III. C. 10. S. 33. Ibid. I
 “ say, had this occurred to his Thoughts,
 “ it is not improbable he would have own-
 “ ed it above all the Pains and Skill he was
 “ master of, to form the above-mentioned
 “ Idea of a Triangle, which is made up
 “ of

“ of manifest, staring contradictions. That
 “ a Man who thought so much, and laid
 “ so great a stress on clear and determinate
 “ Ideas, should nevertheless talk at this rate
 “ seems very surprising.

In this section you plainly accuse Mr. *Locke* of contradicting himself in two several particulars.

1. The above-mentioned Idea of a Triangle, say you, is made up of manifest, staring contradictions.

2. You represent the two following propositions of Mr. *Locke* as contradictory one to the other.

It, The general Idea of a Triangle, is an Idea, wherein some parts of several different and inconsistent Ideas are put together.

Ideas of mixed modes, wherein any inconsistent Ideas are put together, cannot so much as exist in the Mind.

I propose to clear up these two points, and to shew that in neither of them Mr. *Locke* is guilty of contradicting himself: but first, in order thereto, I must take up a little of your time in considering the notion of general, or abstract Ideas. Which pains I am the rather inclined to take because, though I have carefully perused what you have

have written upon this subject, I am one of those who still adhere to the vulgar, or rather universal error of all Mankind, that neither Geometry, nor any other general science can subsist without general Ideas.

Though the words abstract or general Ideas are indifferently used by Writers as having the same common signification; yet as it may be a means of rendering what I have to say upon this subject something more intelligible, I shall beg leave to make a distinction between them, not as being different in themselves, but only in respect of the manner in which they are commonly formed or introduced into the mind.

I shall confine the name of abstract Idea to that, which the mind forms to itself from the consideration of some number of different species, by abstracting from those particular Ideas in which the species differ from one another, and retaining those in which they agree.

I shall call that a general Idea, which may be produced in the mind without any consideration, or even knowledge, of different Species.

An example will make this very plain. When Mr. Ray is forming his Method of
Plants,

Plants, he observes that Mint, and Sage, and Lavender, and Rosemary, and many other Plants, besides their particular characteristics by which they are distinguished from one another, have some other marks in which they all agree; as in their leaves growing in pairs opposite to each other, a monopetalous labiate flower, with four seeds growing at the bottom of it, and those inclosed in no other vessel than the perianthium. By joining together these common marks he forms his compound Idea of that Genus of Plants which he calls verticillate: which from his laying aside, or abstracting from all the peculiar distinguishing marks of the several species, is properly named an abstract Idea.

But if Mr. Ray will teach me Botany by his Method, he must take a different course; he must begin with me where he himself ended; he must first introduce into my mind the general Idea of a verticillate plant, and afterwards descend to particular species. He tells me a verticillate plant is one whose leaves grow in pairs opposite to each other, and whose flower is monopetalous and labiate, with four seeds at the bottom of it, and those inclosed only in the perianthium.

This in me is properly called a general Idea, because I shall find it to comprehend all the particular species of verticillate plants: but I have no reason to call it an abstract Idea, because not knowing as yet any of the particular species, or their characteristick differences, I have nothing to abstract from.

The abstract Idea therefore is that of the Master or Philosopher; and the general Idea that of the Disciple. The former requires, as Mr. *Locke* observes, some pains and skill to form it: the latter demands neither pains nor skill, it needs only a little attention to conceive it.

In like manner if a person acquainted with the several species of Triangles, is from the consideration of these to form an Idea of a Triangle in general; his method will be to examine the several compound Ideas of the different species of Triangles, and to distinguish between such parts of those compound Ideas as are the peculiar characteristicks of each species, and such parts as are common to all of them in general. Then connecting these last together into a new compound Idea, and abstracting from all the rest, he will have the abstract Idea of a Triangle; which is that of a space comprehended

hended by three right lines, add if you please, containing three angles.

When he has got this Idea himself, it is the easiest thing in the world, to give it to another. Let him take a Learner, a Boy, suppose, who has never learned what a triangle is, much less what any particular species of Triangle is, and tell him a Triangle is a space comprehended by three right lines: I say that the Boy, as soon as he understands the meaning of these words, will have acquired the general Idea of a Triangle. If you doubt of it, shew him a rectangular Triangle drawn upon paper, and ask him what it is; he will without hesitation tell you it is a Triangle: afterwards shew him separately all the other species of Triangles, and you will find he knows them every one to be a Triangle. His Idea of a Triangle therefore is general, inasmuch as it suits all the particular species. And the acquiring this Idea either abstract, or general, in Teacher or Scholar, seems to me to be attended with so little difficulty, that I think Mr. *Locke* has said full enough when he declares that the first *requires some pains and skill to form it*: and it is to me surprising to hear a Gentleman of your pene-

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tration profess, that *after reiterated endeavours to apprehend the general Idea of a Triangle, you have found it altogether incomprehensible.* Put your self but once in the case of a Learner, endeavour to divest your mind of all your preconceived Geometrical Ideas, and then turn to *Euclid's* definitions; and I'll venture to assure you, you will find no more difficulty in apprehending the general Idea of a Triangle, than in apprehending the Idea of an obliqueangled, or of a scalene Triangle, or even that of an Angle alone; there being no objection against the first, but what may with equal reason be brought against any of the others; as will easily appear to him that considers, that an angle in general, an obliqueangled Triangle in general, and a scalene Triangle in general can no where exist but in Idea only, any more than a Triangle in general.

Having premised thus much concerning the abstract, or general Idea of a Triangle, I come now to examine into your charge against Mr. *Locke*, and in the first place I must take notice that this charge is introduced in an unfair and unjust manner. *If any one were able to introduce that Idea into my mind; say you, it must be the Author of*
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the Essay concerning Human Understanding; &c. Let us therefore see how this celebrated Author describes the general, or abstract Idea of a Triangle. Would not any body imagine from these words that Mr. Locke were here purposely describing this Idea, in order to introduce it into the mind of one who had it not already? If that were his intention, it is certainly a most miserable description; since no person living who does not already know what a Triangle is, can ever have *that Idea introduced into his mind* from what Mr. Locke here lays down. And yet that Idea is introduced into the mind with all the ease in the world by what he gives us to understand in another * place, that the Idea of a Triangle is that of *three lines, including a space*. Could he possibly talk so clearly in one place, and so cloudily in another, if his intention were the same in both? Is it not plain to any one who attentively reads the passage you refer to, that his intention there was not to *describe* the general Idea of a Triangle, but only to shew from the seeming inconsistencies in that Idea, supposed to be already known, that it required some pains and skill to form it, as well as other

* *Essay on Hum. Underst. B. II. C. 31. S. 6.*

abstract Ideas? Observe his words, " For
 " abstract Ideas are not so obvious or easy
 " to children, or the yet unexercised mind,
 " as particular ones. If they seem so to
 " grown men, 'tis only because by constant
 " and familiar use they are made so. For
 " when we nicely reflect upon them, we
 " shall find, that general Ideas are fictions
 " and contrivances of the mind, that carry
 " difficulty with them, and do not so easily
 " offer themselves, as we are apt to ima-
 " gine. For example, Does it not require
 " some pains and skill to form the general
 " Idea of a Triangle? (Which yet is none
 " of the most abstract, comprehensive and
 " difficult.) For it must be neither oblique,
 " nor rectangle, neither equilateral, equicru-
 " ral, nor scalenon; but all and none of
 " these at once. In effect, it is something
 " imperfect, that cannot exist; an Idea where-
 " in some parts of several different and in-
 " consistent Ideas are put together."

We come now to the manifest, staring
 contradictions, contained in this Idea of a
 Triangle: the first of which, I suppose, is
 contained in these words, *all and none of these*
at once. The *Enantiosis*, I confess, is pretty
 strong: and yet the meaning of it is plain-
 ly

ly no more than this, that the general Idea of a Triangle is a part of the Idea of every species of Triangles here enumerated, but is not the intire Idea of any one of them; is common to them all, and confined to none. *It is something imperfect that cannot exist*, may possibly be another of your contradictions. It does not appear so to me. For every individual Triangle, every Triangle that can exist, must be something more than a space included by three lines, it must also have the characteristick mark of some one of the particular species of Triangles; without which it would be *imperfect, it could not exist*, which is what Mr. Locke here says of a Triangle in general.

2. But the great contradiction of all seems to lie in the two following propositions, which are brought together from different parts of Mr. Locke's works, and set to stare one another in the face to disgrace their Author.

It is an Idea, wherein some parts of several different and inconsistent Ideas are put together.

Ideas of mixed modes, wherein any inconsistent Ideas are put together, cannot so much as exist in the mind.

Here,

Here, Sir, I strongly apprehend you are fallen into one of those traps, which this Great Man would sometimes divert himself with setting to catch unwary cavillers, the *Homines stolidos & ad depugnandum paratos*, that I mentioned a while ago. Had his first proposition run thus, It is an Idea, wherein several different and inconsistent Ideas are put together, it would undoubtedly have been contradictory to the second. But that is not the case : pray observe the words of this cautious and accurate Writer. It is an Idea, wherein **SOME PARTS OF** several different and inconsistent Ideas are put together. Now, we know that the several compound Ideas of a rectangled, an oblique, and an acuteangled Triangle are different and inconsistent one with another. No two of them can be put together so as jointly to exist or be conceived in the mind. Likewise the several compound Ideas of an equilateral, equicrural, and scalene triangle are inconsistent with one another. But yet some parts of one of these inconsistent Ideas are not only consistent, but are perfectly the same with some parts of another. To shew this I beg leave to divide two of these inconsistent Ideas into several parts.

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The compound Idea of a rectangled Triangle may be divided into these parts.

1. A plain space,
2. Comprehended by right lines,
3. Three in number,
4. Containing three angles,
5. One right, two acute.

The compound Idea of an acuteangled Triangle may be divided into these parts.

1. A plain space,
2. Comprehended by right lines,
3. Three in number,
4. Containing three angles,
5. All acute.

There is, we see, no difference between the four first parts of the compound Idea of a rectangled Triangle, and the four first parts of that of an acuteangled Triangle: it is owing to the fifth part alone of each Idea, that these two Ideas are different and inconsistent. And as it is easy to see that these four first parts are the same in all the other particular Species of Triangles; and that the same four parts do compose the general Idea of a Triangle; it is plain that the general Idea of a Triangle is an Idea, wherein SOME PARTS OF several different and inconsistent Ideas are put together.

The first therefore of the two propositions
in

in question is undoubtedly true; and as these parts are no way inconsistent with one another, it is manifest that the second proposition is not contradictory, or at all repugnant to the first.

I come now, Sir, to take my leave of you, and hope that if an honest zeal for truth in the first place, and in the second for the reputation of those Gentlemen to whom I conceive the whole body of mankind, at least I must acknowledge my self to be highly indebted, has given occasion not only of differing from you, but even of reprehending you with the utmost freedom wherever I thought the truth and your behaviour required it; you will not impute the liberty I have taken to any disrespect for your person, which I am an utter stranger to, though I have a very great esteem and value for your uncommon abilities and many of your writings, and am with sincere respect,

SIR,

Your most obedient

Humble Servant,

PHILALETHES CANTABRIGIENSIS.

F I N I S.



